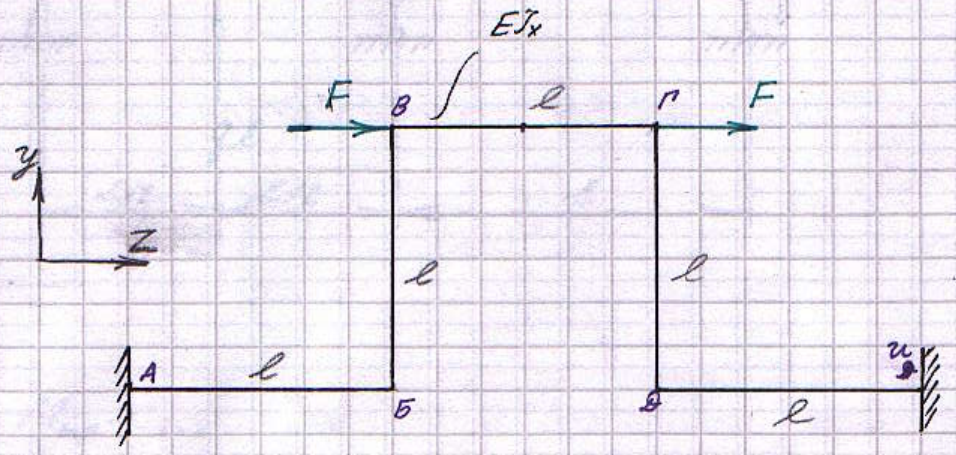
1 Рама, не имеющая замкнутых контуров (построить эпюры из. моментов):



Решение

I. Вычисление степени статической неопределимости:

а) $n_{внеш.\,об} = \underset{А}{3} + \underset{Д}{3} = 6$

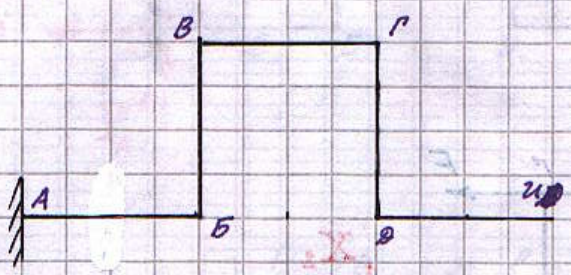
б) $n_{внутр.\,св.} = 3 \cdot 0 = 0$

з) $n = 6 + 0 - 3 = 3$

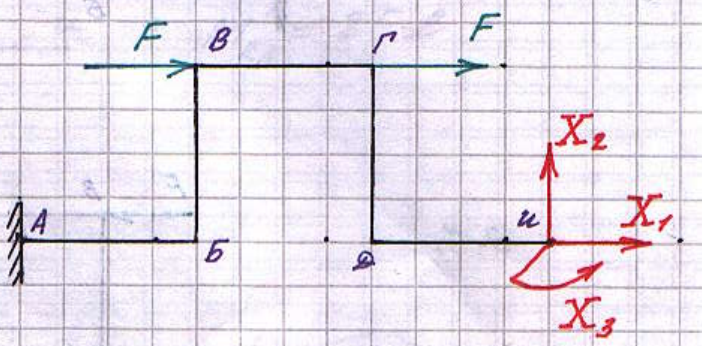
II. Раскрытие статической неопределимости

а)

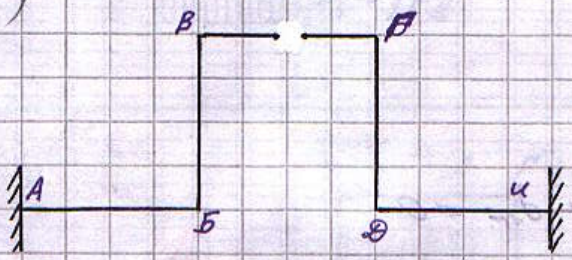
1) о.с. №1



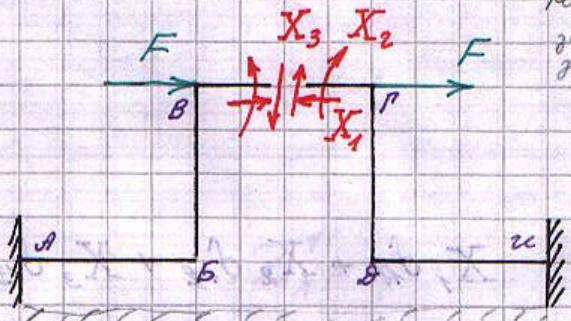
э.с. №1



3) !!! о.с. №3



э.с №3

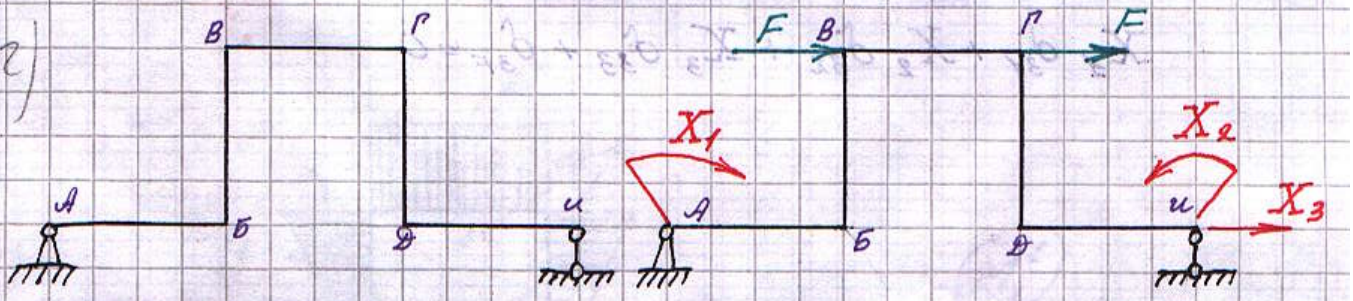


Замкнутый контур – замкнут весь

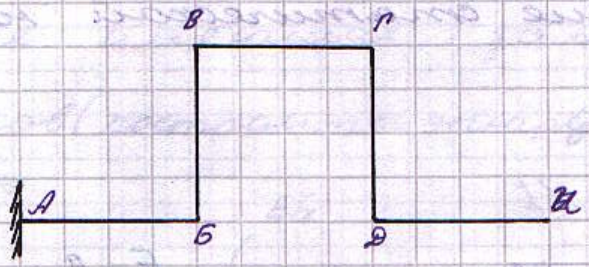
о.с. №2

э.с №2

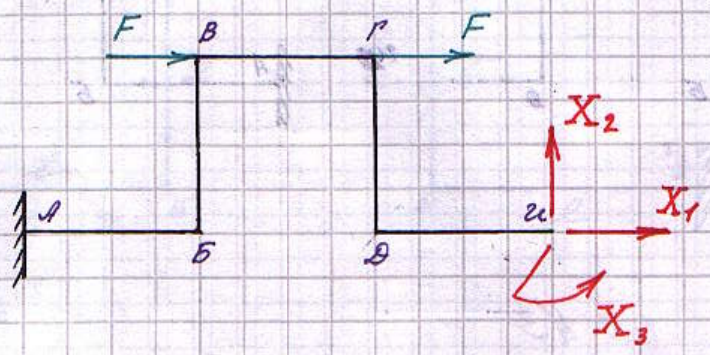
г)



б) О.С.1



Э.С.



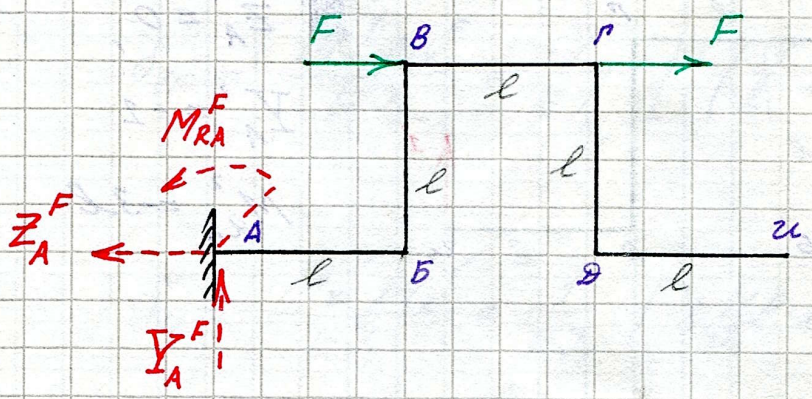
в)

$$X_1 \delta_{11} + X_2 \delta_{12} + X_3 \delta_{13} + \delta_{1F} = 0$$

$$X_1 \delta_{21} + X_2 \delta_{22} + X_3 \delta_{23} + \delta_{2F} = 0$$

$$X_1 \delta_{31} + X_2 \delta_{32} + X_3 \delta_{33} + \delta_{3F} = 0$$

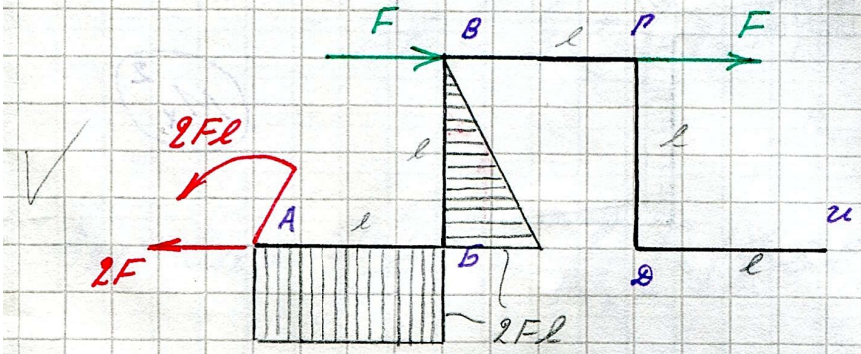
г)



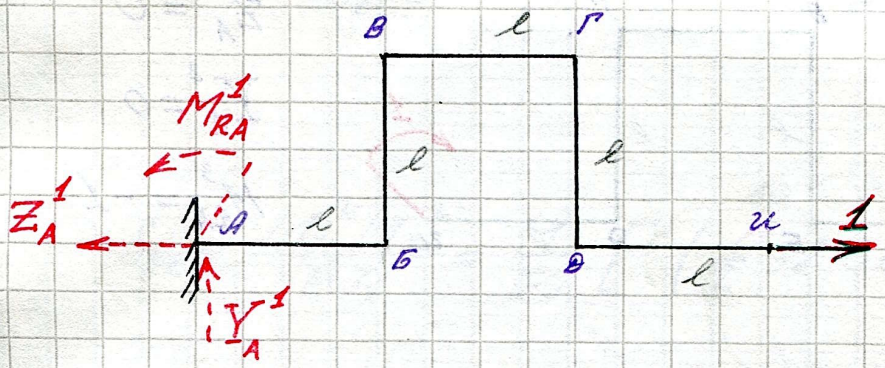
$Z_A^F = 2F$

$Y_A^F = 0$

$M_{RA}^F = 2F \cdot \ell$



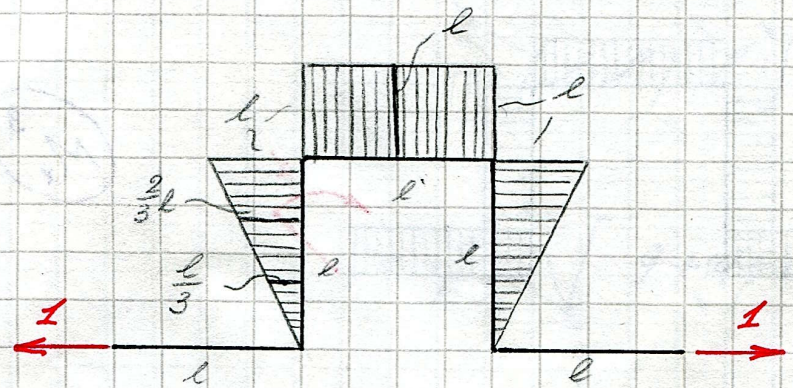
M_x^F



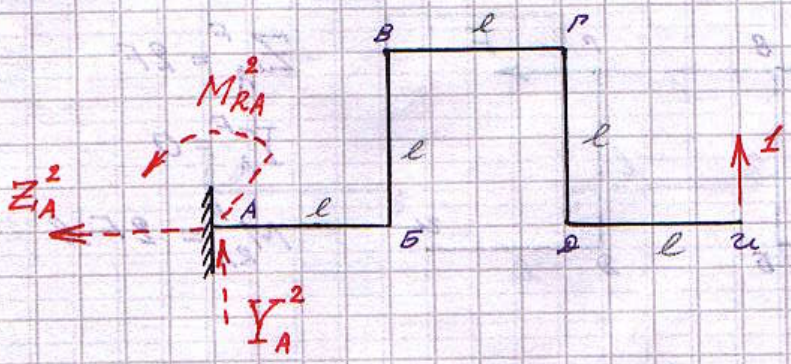
$Z_A^1 = 1$

$Y_A^1 = 0$

$M_{RA}^1 = 0$



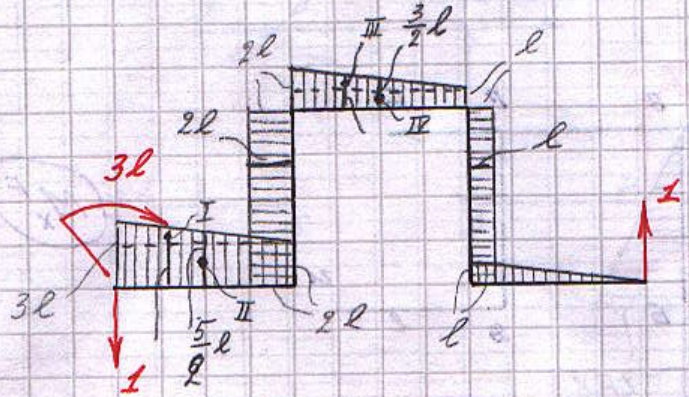
M_x^1



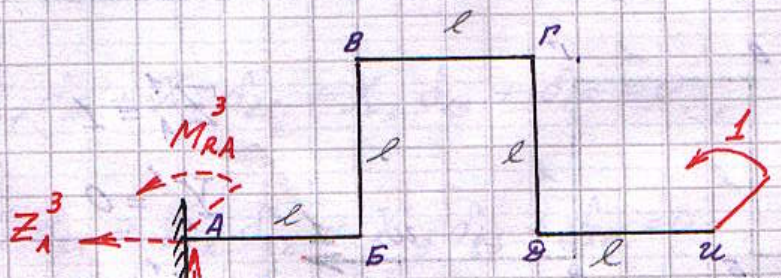
$Z_A^2 = 0$

$Y_A^2 = -1$

$M_{RA}^2 = -3\ell$



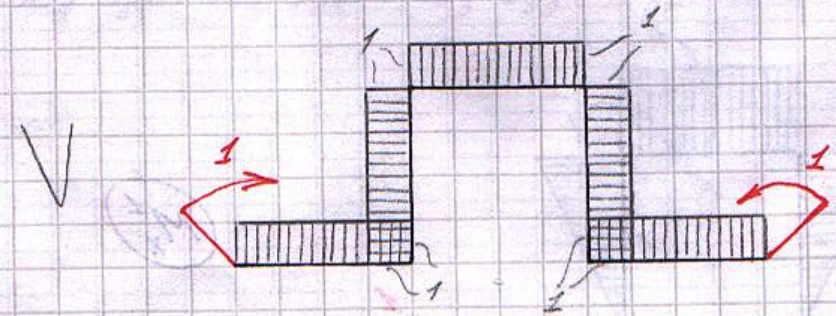
M_x^2



$Z_A^3 = 0$

$Y_A^3 = 0$

$M_{RA}^3 = -1$



M_x^3

$$\delta_{1F} = \frac{M_x^1 M_x^F}{EJ_x} = \frac{-1}{EJ_x}\cdot\left[\left(\frac{1}{2}\cdot 2Fl\cdot l\right)\cdot\frac{l}{3}\right] = -\frac{Fl^3}{3EJ_x} \quad 2{,}66$$

$$\delta_{11} = \frac{M_x^1 M_x^1}{EJ_x} = \frac{1}{EJ_x}\cdot\left[2\left(\frac{1}{2}l\cdot l\right)\cdot\frac{2}{3}l + (l\cdot l)\cdot l\right] =$$

$$= \frac{l^3}{EJ_x}\left[\frac{2}{3}+\frac{3}{3}\right] = \frac{5}{3}\cdot\frac{l^3}{EJ_x} \quad 4{,}667$$

$$\delta_{21} = \delta_{12} = \frac{M_x^1 M_x^2}{EJ_x} = \frac{1}{EJ_x}\left[\left(\frac{1}{2}l\cdot l\right)\cdot 2l + (l\cdot l)\frac{3}{2}l + \left(\frac{1}{2}ll\right)\cdot l\right] =$$

$$= \frac{l^3}{EJ_x}\left[1+\frac{3}{2}+\frac{1}{2}\right] = \frac{3l^3}{EJ_x} = \frac{9l^3}{3EJ_x}$$

(3)

все поп. отрезки

$$\delta_{22} = \frac{M_x^2 M_x^2}{EJ_x} = \frac{1}{EJ_x}\left[\left(\frac{1}{2}\cdot 3l\cdot 3l\right)\cdot\frac{2}{3}\cdot 3l + (2l\cdot l)\cdot 2l +\right.$$

$$\left. + (l\cdot l)\cdot l\right] =$$

$$= \frac{l^3}{EJ_x}\cdot\left[9+4+1\right] = \frac{14l^3}{EJ_x} = \frac{42}{3}\cdot\frac{l^3}{EJ_x}$$

16

$$\delta_{33} = \frac{M_x^3 M_x^3}{EJ_x} = \frac{1}{EJ_x}\left[(1 \cdot 5\ell)\cdot 1\right] = \frac{5\ell}{EJ_x} = \frac{15}{3}\cdot\frac{\ell}{EJ_x}$$

(5)

$$\delta_{13} = \delta_{31} = \frac{M_x^1 M_x^3}{EJ_x} = \frac{1}{EJ_x}\cdot\left[\frac{1}{2}\left(\frac{1}{2}\ell\,\ell\right)\cdot 1 + (\ell\cdot\ell)\cdot 1\right] =$$

$$= \frac{\ell^2}{EJ_x}\cdot[1+1] = \frac{2\ell^2}{EJ_x} = \frac{6\ell^2}{3EJ_y}$$

(2)

$$\delta_{23} = \delta_{32} = \frac{M_x^3 M_x^2}{EJ_x} = \frac{1}{EJ_x}\left[\left(\frac{1}{2}3\ell\,3\ell\right)\cdot 1 + (2\ell\,\ell)\,1 + \right.$$

$$\left. + (\ell\cdot\ell)\cdot 1\right] =$$

$$= \frac{\ell^2}{EJ_x}\left[\frac{9}{2} + \frac{4}{2} + \frac{2}{2}\right] = \frac{15}{2}\,\frac{\ell^2}{EJ_x} =$$

$$= \frac{22{,}5}{3}\cdot\frac{\ell^2}{EJ_x}$$

(7,5)

$$\delta_{2F} = \frac{M_x^2 M_x^F}{EJ_x} = \frac{1}{EJ_x}\left[-\left(\frac{1}{2}Fl\cdot l\right)\frac{5}{3}l - \left(\frac{1}{2}\cdot\frac{1}{2}Fl\cdot l\right)\cdot 2l\right] =$$

$$= \frac{Fl^3}{EJ_x}\left[-5-2\right] = -\frac{7Fl^3}{EJ_x} = -\frac{21Fl^3}{3EJ_x}$$

-7

$$\delta_{3F} = \frac{M^3 M^F}{EJ_x} = \frac{1}{EJ_x}\left[-\left(2Fl\cdot l\right)\cdot 1 - \left(\frac{1}{2}\cdot 2Fl\cdot l\right)\cdot 1\right] =$$

$$= -\frac{Fl^2}{EJ_x}\left[-2-1\right] = -\frac{9Fl^2}{3EJ_x}$$

-3

9)

Итак:

$$\frac{l}{3EJ_x} \cdot \begin{cases} X_1 \cdot 5l^2 + X_2 \cdot 9l^2 + X_3 \cdot 6l = Fl^2 \\ X_1 \cdot 9l^2 + X_2 \cdot 42l^2 + X_3 \cdot 22{,}5l = 21Fl^2 \\ X_1 \cdot 6l + X_2 \cdot 22{,}5l + X_3 \cdot 15 = 9Fl \end{cases}$$

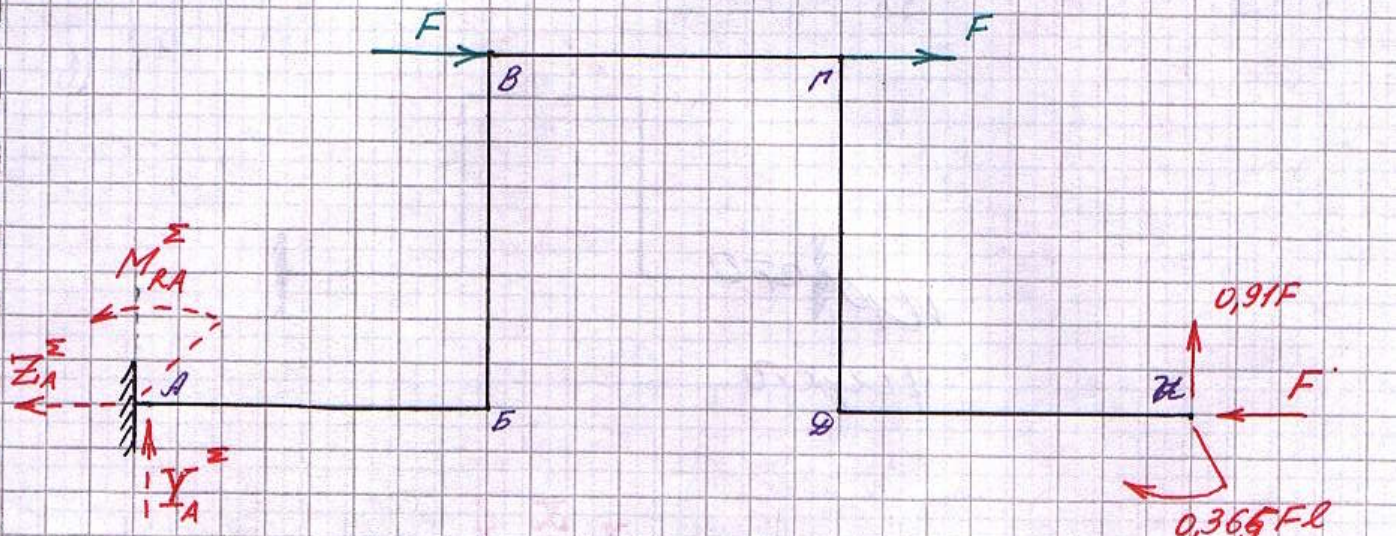
$$\frac{l}{3EJ_x} \cdot \left(\begin{bmatrix} 5l^2 & 9l^2 & 6l \\ 9l^2 & 42l^2 & 22{,}5l \\ 6l & 22{,}5l & 15 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} l \\ 21l \\ 9 \end{Bmatrix} \cdot F \cdot l \right)$$

$X_1 = -F$

$X_2 = 0{,}91 \cdot F$

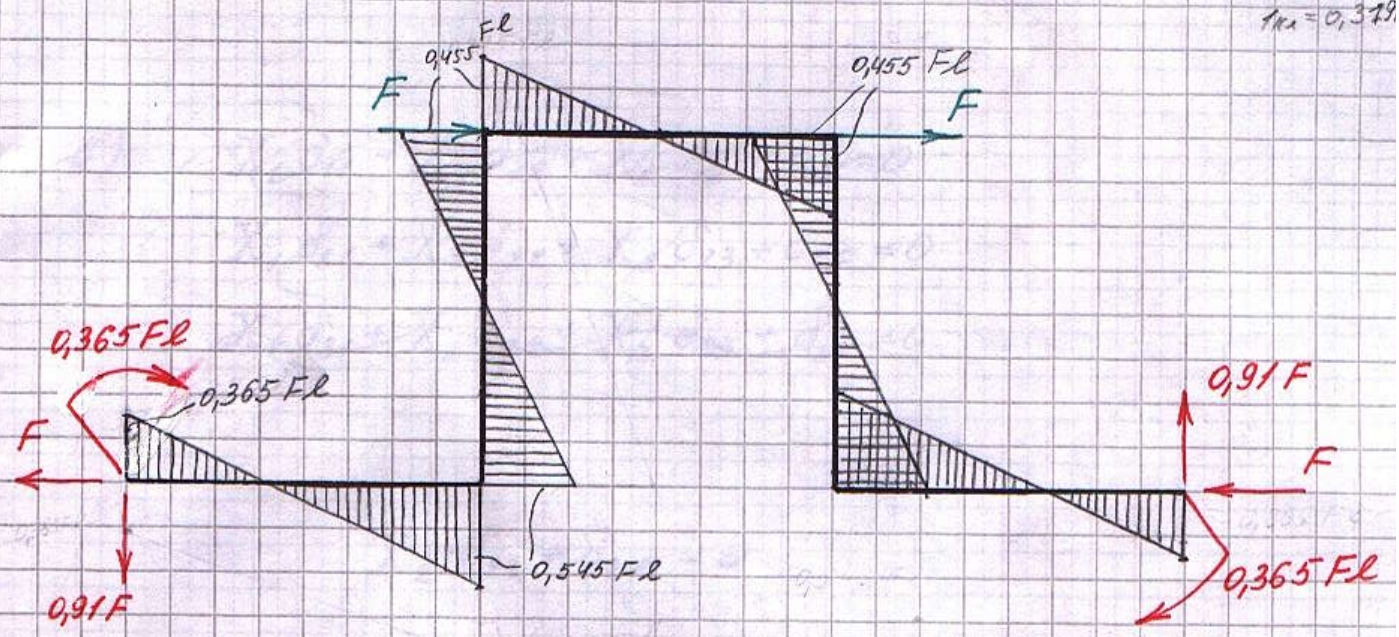
$X_3 = -0{,}365 \cdot F \cdot l$

e)



$$\sum M_A = 0 = M^{\Sigma}_{RA} + \overbrace{0{,}91 \cdot F \cdot 3\ell}^{2{,}73F} - 2F\ell - 0{,}365F\ell \Rightarrow M^{\Sigma}_{RA} = -0{,}365F$$

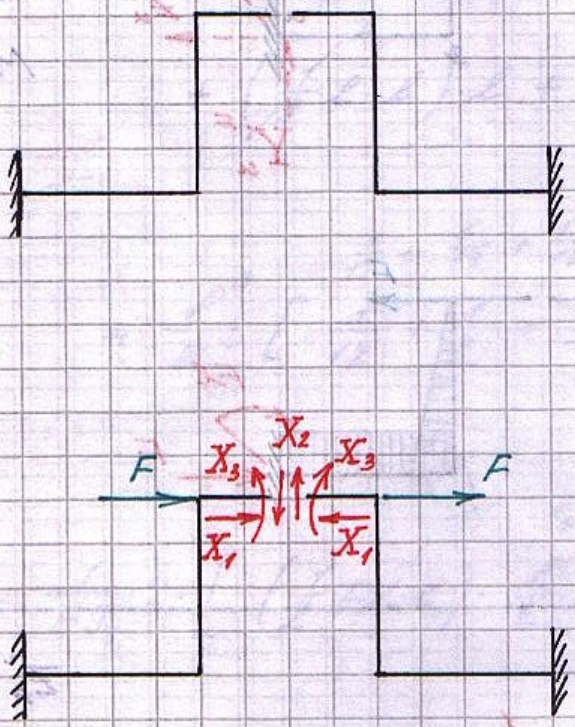
$$Y^{\Sigma}_{A} = -0{,}91 \cdot F \quad ; \quad Z^{\Sigma}_{A} = F$$



IV. Проверка

0, с.

б)



$X_{41} = 0$ – косая симметр

$X_3 = 0$

в) ~~$X_1 \delta_{11} + X_2 \delta_{12} + X_3 \delta_{13} + \delta_{1F} = 0$~~

$X_1 \delta_{21} + X_2 \delta_{22} + X_3 \delta_{23} + \delta_{2F} = 0$

~~$X_1 \delta_{31} + X_2 \delta_{32} + X_3 \delta_{33} + \delta_{3F} = 0$~~

$$\Downarrow$$

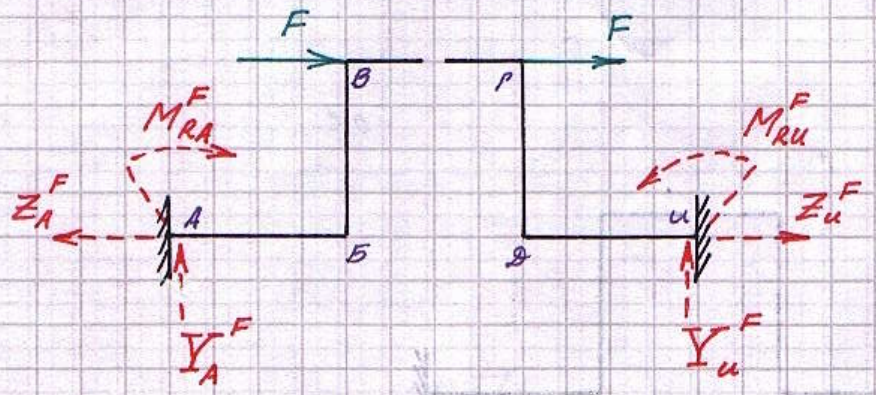
$$X_2 \cdot \delta_{22} + \delta_{2F} = 0$$

2)

$Z_A^F = F$

$Y_A^F = 0$

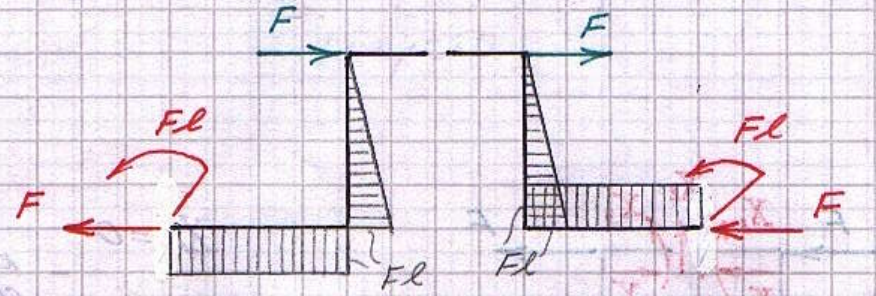
$M_{RA}^F = -Fl$



$Z_u^F = -F$

$Y_u^F = 0$

$M_{Ru}^F = Fl$



M_x^F

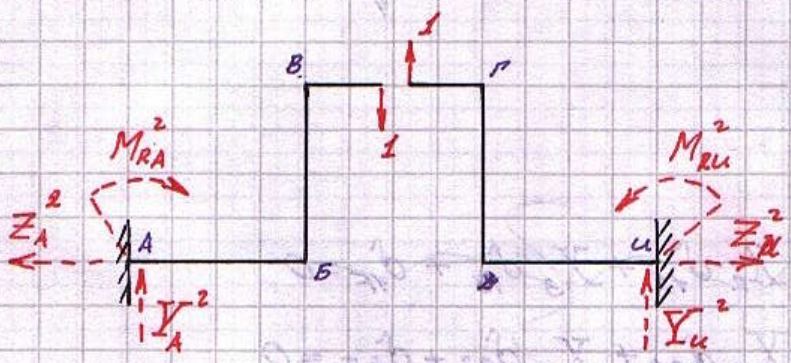
$Z_A^2 = Z_u^2 = 0$

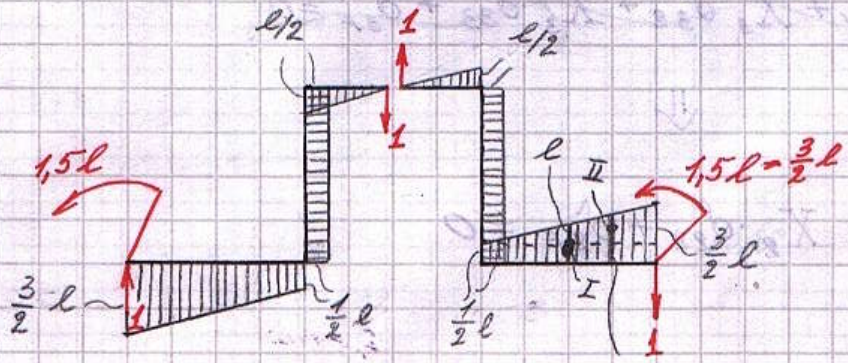
$Y_A^2 = 1$

$Y_u^2 = -1$

$M_{RA}^2 = -1{,}5l$

$M_{Ru}^2 = 1{,}5l$





M_x^2

$$\delta_{22} = \frac{2}{EJ_x}\cdot\left[\left(\frac{1}{2}\cdot\frac{1}{2}\ell\cdot\frac{1}{2}\ell\right)\cdot\frac{2}{3}\cdot\frac{1}{2}\ell + \left(\frac{1}{2}\ell\cdot\ell\right)\cdot\frac{1}{2}\ell + \right.$$

$$\left. + \left(\frac{1}{2}\ell\cdot\ell\right)\cdot\ell + \left(\frac{1}{2}\ell\cdot\ell\right)\cdot\underbrace{\left(\frac{31}{32}\ell + \frac{22}{23}\ell\right)}_{\frac{7}{6}\ell}\right] =$$

$$= \frac{\ell^3}{EJ_x}\left[\underbrace{\frac{1}{12} + \frac{5}{12} + \frac{12}{12} + \frac{14}{12}}_{1} + \frac{7}{6}\right] = \frac{33}{12}\cdot\frac{\ell^3}{EJ_x}$$

$$\delta_{2F} = \frac{1}{EJ_x}\cdot 2\cdot\left[\left(\frac{1}{2}F\ell\cdot\ell\right)\cdot\frac{\ell}{2} + \left(F\ell\cdot\ell\right)\cdot\ell\right] =$$

$$= \frac{F\cdot\ell^3}{EJ_x}\cdot\left[\frac{1}{2} + \frac{4}{2}\right] = \frac{5}{2}\cdot\frac{F\ell^3}{EJ_x}$$

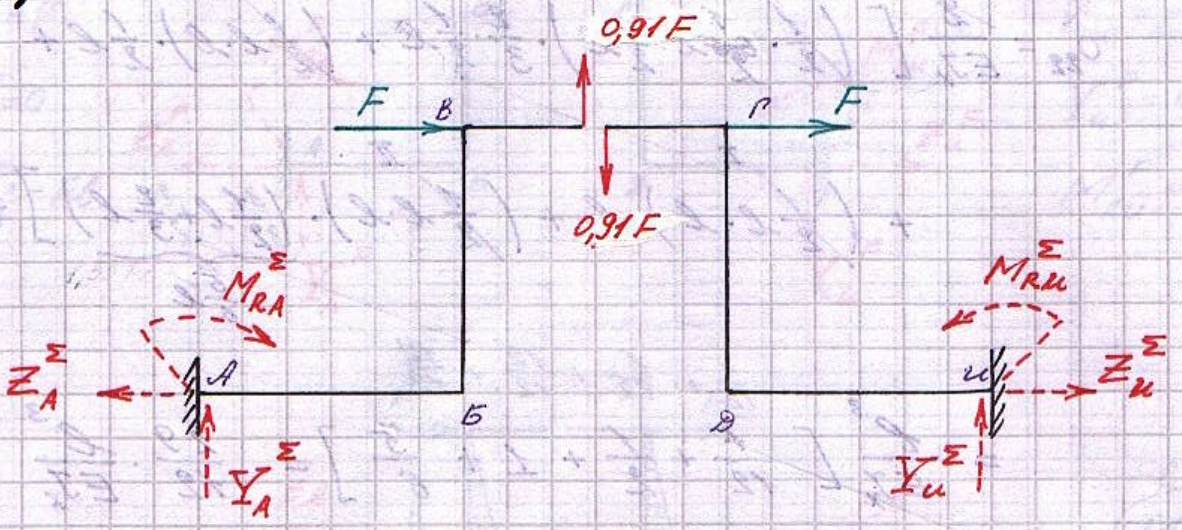
g)

$$X_2\,\delta_{22} + \delta_{2F} = 0$$

$$X_2 = -\frac{\delta_{2F}}{\delta_{22}} = -\frac{\frac{5}{2}\cdot\frac{F\ell^3}{EJ_x}}{\frac{33}{12}\cdot\frac{\ell^3}{EJ_x}} = -\frac{5}{2}\cdot\frac{12}{33}F =$$

$$= -\frac{30}{33}F = -0{,}91\cdot F$$

e)



$Z_A^{\Sigma} = F,$

$Z_Д^{\Sigma} = -F$

$Y_A^{\Sigma} = -0,91F$

$Y_Д^{\Sigma} = 0,91F$

$M_{RA} = +0,365 \cdot F\ell$; $M_{RД} = -0,365\ F\ell$